12^{th} Annual Johns Hopkins Math Tournament
Saturday, February 19, 2011

General Test 2

1. [**1025**] Let $a, b \in \mathbb{C}$ such that $a + b = a^2 + b^2 = \frac{2\sqrt{3}}{3}i$. Compute $|\text{Re}(a)|$.

2. [**1026**] You are given a dart board with a small circle that is worth 20 points and a ring surrounding the circle that is worth 11 points. No points are given if you do not hit any of these areas. What is the largest integeral number of points that cannot be achieved with some combination of hits.

3. [**1028**] Compute the largest value of r such that three non-overlapping circles of radius r can be inscribed in a unit square.

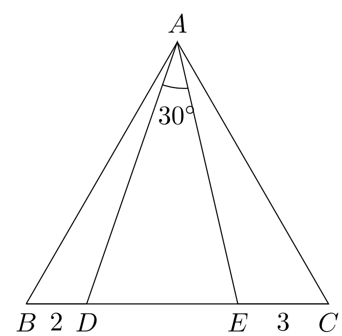
4. [**1032**] Find the ten smallest x, with $x > 1$, that satisfy the following relation:

$$\sin(\ln x) + 2\cos(3\ln x)\sin(2\ln x) = 0$$

5. [**1040**] If r, s, t, and u denote the roots of the polynomial $f(x) = x^4 + 3x^3 + 3x + 2$, find

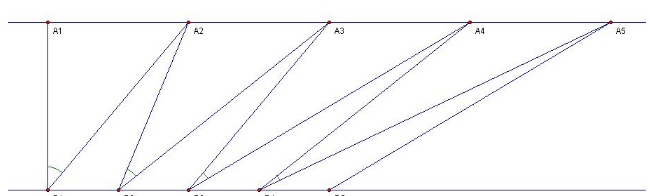
$$\frac{1}{r^2} + \frac{1}{s^2} + \frac{1}{t^2} + \frac{1}{u^2}$$

6. [**1056**] Let $\triangle ABC$ be equilateral. Two points D and E are on side BC (with order B, D, E, C), and satisfy $\angle DAE = 30°$. If $BD = 2$ and $CE = 3$, what is BC?



7. [**1088**] Two ants, Yuri and Jiawang, begin on opposite corners of a cube. On each move, they can travel along an edge to an adjacent vertex. Find the probability they both return to their starting position after 4 moves.

8. [**1152**] Two parallel lines ℓ_1 and ℓ_2 are on a plane with distance d. On ℓ_1 there are infinitely many points $A_1, A_2, A_3, \cdots$ progressing in the same distance: $A_nA_{n+1} = 2$ for all n. In addition, on ℓ_2 there are also infinite points $B_1, B_2, B_3, \cdots$ satisfying $B_nB_{n+1} = 1$ for all n. Given that A_1B_1 is perpendicular to both ℓ_1 and l_2, express the sum $\sum_{i=1}^{\infty} \angle A_iB_iA_{i+1}$ in terms of d.



9. [**1280**] Let $\{a_i\}_{i=1,2,3,4}, \{b_i\}_{i=1,2,3,4}, \{c_i\}_{i=1,2,3,4}$ be permutations of $\{1, 2, 3, 4\}$. Find the minimum of $a_1b_1c_1 + a_2b_2c_2 + a_3b_3c_3 + a_4b_4c_4$.

10. [**1536**] How many functions f that take $\{1, 2, 3, 4, 5\}$ to itself, not necessarily injective or surjective, satisfy $f(f(f(x))) = f(f(x))$ for all x in $\{1, 2, 3, 4, 5\}$?